Matching Socks

You are about to leave for holiday, but you forgot socks! You race back to your room, but all the lights are off, so you can't see the colour of the socks.

Never mind, because you remember that in your drawer there are ten pairs of white socks, ten pairs of black socks, and eleven pairs of blue socks, but they are all mixed up.

How many of your socks do you need to take before you can be sure to have at least one matching pair?

Find the value of the letters

ABC × DEF = 123456, if A = 1

Absolutely Christmas Crackers

Last month I sent off for one of those kits which you can use to make your own Christmas Crackers. The kit contained:

Three colours of hat: Red, Yellow and Blue

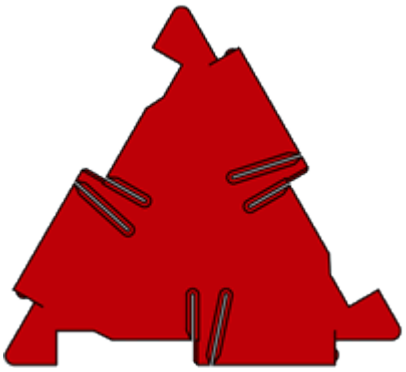
Four types of novelty: toy car, spinning top, magnifying glass and miniature hair brush

Four different types of joke slip

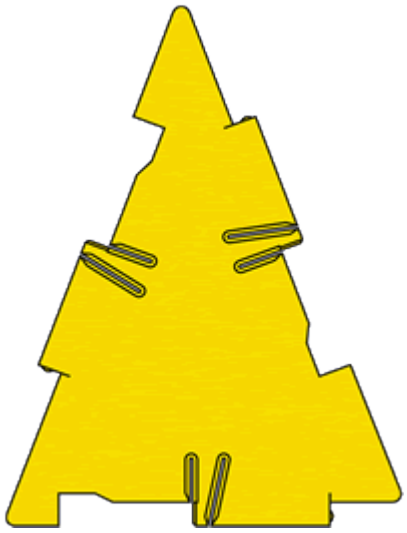
All the other parts were the same type. The kit contained enough bits for 50 crackers. Can I make each cracker different from all the others?

Triangles to Tetrahedra

A tetrahedron is a three dimensional shape with four faces. Each face is a triangle. Here are four different triangles made from Polydron (plastic shapes which click together):



Small equilateral triangle



Isosceles triangle

Right angled isosceles triangle

Large equilateral triangle

The sides of the small equilateral triangle are the same length as the short side of the isosceles triangle and the short sides of the right-angled isosceles triangle.

The sides of the large equilateral triangle are the same length as the long sides of the isosceles triangle and the long side of the right-angled isosceles triangle.

You have an unlimited number of each type of triangle.

How many different tetrahedra can you make? Convince yourself you have found them all.

Consecutive Negative Numbers

Do you notice anything about the solutions when you add and/or subtract consecutive negative numbers?

Take, for example, four consecutive negative numbers, say

−7, −6, −5, −4

Now place + and/or − signs between them.

e.g.

−7 + −6 + −5 + −4

−7 − −6 + −5 − −4

There are other possibilities. Try to list all of them.

Now work out the solutions to the various calculations.

e.g.

−7 + −6 + −5 + −4 = −22

−7 − −6 + −5 − −4 = −2

Choose a different set of four consecutive negative numbers and repeat the process.

Take a look at both sets of solutions. Notice anything?

Can you explain any similarities?

Can you predict some of the solutions you will get when you start with a different set of four consecutive negative numbers?

Test out any conjectures you may have.

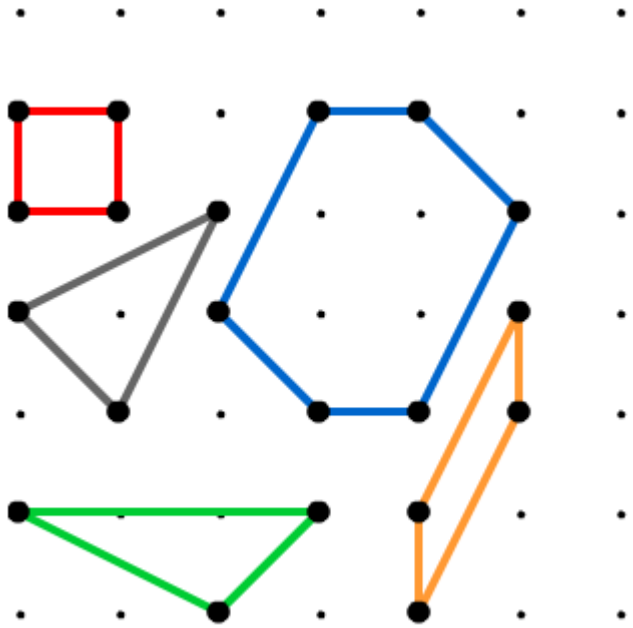
Try to explain and justify your findings.

Pick's Theorem

When the dots on square dotty paper are joined by straight lines the enclosed figures have dots on their perimeter (*p*) and often internal (*i*) ones as well.

Figures can be described in this way: (*p*,*i*).

For example, the red square has a (*p*,*i*) of (4,0), the grey triangle (3,1), the green triangle (5,0) and the blue shape (6,4):



Each figure you produce will always enclose an area (A) of the square dotty paper.

The examples in the diagram have areas of 1, 1½, and 6 square units.

Do you agree?

Draw more figures; tabulate the information about their perimeter points (*p*), interior points (*i*) and their areas (A).

Can you find a relationship between all these three variables (*p*, *i* and A)?